

Stability and Control Analysis of Centrifugal Compressor

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ABSTRACT

Compressor stall and surge are complex nonlinear instabilities that reduce the performance and can cause failure of the system. The emphasis has been on gaining a better insight to stall/surge dynamics and to predict their occurrence and controlling. This report presents a bifurcation-based analysis of the dynamical behavior and control of stall and surge in centrifugal compressor stage using MG86 model. This analysis gives compressor instabilities and provides a complete picture of the flow behavior at different operating points. A proposed nonlinear control law by Gu [9] along with the effect of rotational disturbances has been studied using this method. Using this control law it was shown that, abrupt entry of compressor into rotating stall can be avoided, eliminating the hysteresis between entry into and recovery from the rotating stall. Through analysis it was shown that system maintains stability up to a certain value of compressor rotational speed.

NOMENCLATURE

ϕ = Experimental flow coefficient averaged over angle (axial velocity divided by wheel speed)
 ψ = Experimental total to static pressure rise coefficient (inlet to plenum)
 ψ_{co} = Shut-off value of axisymmetric characteristic before rescaling
 Φ = Flow coefficient averaged over angle (axial velocity divided by wheel speed) rescaled with W.
W = Half of the mass flow, which gives maximum pressure rise
 Ψ = Total to static pressure rise coefficient (inlet to plenum) rescaled with H
 Ψ_{co} = Shut-off value of axisymmetric characteristic after rescaling
H = Half the difference between maximum pressure rise and the shut off head
 Ψ_c = Axisymmetric pressure rise coefficient rescaled with H (response of compressor in steady axisymmetric flow)
 Ψ_r = Rotating stall characteristic after rescaling
 $\beta = 2 B S$
 $B = (U/2a_s) (V_p/A_c L_c)^{1/2}$
 a_s = Speed of sound
 V_p = Volume of plenum
 L_c = Effective length of compressor in wheel radii
 $S = H / W$
 Φ_T = Throttle characteristic
R = Square of the amplitude of first mode of rotating stall disturbances
 ξ = Non-dimensional time variable
 σ = Parameter reflecting effective length of compressor taken as 7
K = Controller gain parameter

SUBSCRIPTS

0 new nominal value under closed loop conditions
e equilibrium point
eff effective value

SUPERSCRIPT

Derivative with respect to time

1. INTRODUCTION

Reduction in mass flow rate results in the pressure rise and increases the power output but only to a point, beyond which the steady flow is no longer stable. One would like to operate the compressor as

close as possible to this critical point, but in such cases, a small perturbation in the flow may be enough to push the compressor into the unstable region. Stable operations of Centrifugal compressors

are constrained by two aerodynamic flow instabilities: rotating stall and surge. Several experimental investigations have been focused on verifying that in fact compressors exhibit long-length perturbations, which resonate and subsequently grow into rotating stall. The first of these conducted by McDougall [1], who showed that circumferentially rotating sinusoids of axial velocity perturbation preceded rotating stall inception in a low speed compressor. Garnier [2] was able to measure these waves in low speed as well as high-speed compressors. Separation of flow from blade surface, which generally occurs at an increased incidence, is known as stalling. The mechanism for stall inception has been well explained by day [3].

Stall inception does not occur until the local mass flow through the blades is, at some spatial location, low enough that a small region of the annulus experiences catastrophic separation in the blade passage [4]. The spike or pip that develops is governed by separation and three dimensional flow distribution phenomena [3], [4], [5], both of which are nonlinear. Rotating stall is characterized as the periodic break down of Energy transfer from the rotor to the flow with in a portion of the impeller channel at any given time [6]. At initiation of stall, the impeller goes into rotating stall. When the compressor goes in to rotating stall, depending up on the severity of stall flow, rotate around the annulus of the compressor at about 20% to 70% of the rotor speed. Rotating with such a cell in a frame of reference, to an observer, the flow is steady. In a relative frame of reference the stall cells rotate at opposite to the direction of rotation. Loading and unloading the impeller blades occurs as they pass in and out of the stall cells. The presence of stall cells creates circumferential asymmetry in the flow through the compressor and induces large vibratory stresses in the blading and depending on the stall characteristic can result in a large drop in performance and efficiency.

In a centrifugal compressor stage either impeller or diffuser can stall depending on the incidence. Stalling of any one component will influence the other component. The rotating stall cells at impeller outlet will propagate both upstream and down stream influencing the flow in diffuser as well as the impeller inlet flow. Initially rotating stall is assumed to affect only one blade passage and large stall cell is established covering about entire impeller of the compressor. The frequency of the rotating stall depends on the speed with which it rotates and the number of rotating stall cells. The transition from normal compressor operation into rotating stall and from rotating stall to normal compressor operation has been well explained in [8], [9] which is known as hysteresis in rotating stall.

Surge is characterized by large amplitude fluctuation of the pressure rise and unsteady (but circumferentially uniform) annulus-averaged mass flow. Operation during surge results in considerable loss performance and efficiency and can lead to high blade and casing vibrations.

Depending on the amplitude of flow and pressure fluctuations, four categories of surge can be distinguished [10], [11]: mild surge, classic surge, modified surge, and deep surge. Classic surge is a two-dimensional nonlinear phenomenon with larger oscillations and at a lower frequency than mild surge but the mass flow fluctuations remain positive. It is that type of oscillations where rotating stall supposed to be there during part of the cycle. Deep surge is generally assumed to be one-dimensional flow in which reverse flow over part of the cycle occurs. The frequency of oscillations during deep surge is normally well below the Helmholtz frequency [10].

The first mathematical model to explain the instabilities surge, stall phenomena for low speed (incompressible) flow was given by Greitzer [12]. Moore and Greitzer (1986) [7] extended the previous model to describe surge and rotating stall phenomena in axial flow compression systems. Most of the theoretically developed surge & stall analysis and control strategies have been based on the one-mode approximation of the Moore-Greitzer known as MG86 model.

A detailed analysis of the MG86 model has been carried out by McCaughan [13], [14], [15] using bifurcation method. The analysis of MG86 model carried out by Agarwal, Ananthkrishnan [16] and also by Ananthkrishnan, Vaidya and Walimbe [17], contradicts the analysis made by McCaughan [13], [14]. They showed that the onset of deep surge limit cycles is independent of the presence of classic surge limit cycles. Miroslav Krstic, Dan Fontaine [18] approached Bifurcation control using Lyapunov tools, who claims that their controller achieves not only locally but also globally. Liaw and Abed [19] developed a nonlinear controller that changes the character of the bifurcation at the stall inception point, from hard sub critical to soft supercritical, thus avoiding an abrupt transition into rotating stall.

Most of the developments in the field of compressor flow stability and active control over the last two decades have been reviewed by paduano [19]. Willems and De Jager [10] give a list of existing compressor flow models and their properties. An overview of stall/surge dynamics and control techniques applied to axial flow compressors has been available in [21]. Chaoqun Nie, Gang Xu, Xiaobin Chang, Jingyi Chen. [22] Approached steady micro air injection from the casing having an idea to improve the stability of a three-stage low speed axial compression system. Unsteady responses

are demonstrated through dynamic signal analysis using a wavelet-based method to show the behavior of early flow disturbances under the influence of injection. Greitzer [23], conducted experiments on a three-stage axial flow compressor. In his report he has explained the axial compressor stall and surge using a non-dimensional parameter B. He observed that the deep surge mode has a large amount of hysteresis associated with its onset and cessation.

In the literature [24], the limits of Rotating stall and stall in vane less diffuser of centrifugal compressors have been explained. In this literature it is said that when rotating stall occurs in a vane less diffuser, the wall friction force, as well as the exit loss of the kinetic energy, varies with respect to time due to rotation of the non-axisymmetric distribution of the velocity, but the time mean value do not change much.

Ramamurthy. S, and Murugesan. K [25], [26] carried out experiments to measure the flow field at the impeller outlet using hot-wire sensor placed very closed to the impeller tip. They described experimental method to estimate the stall propagating speed and the number of stall cells. They concluded that at initiation of stall the impeller goes into rotating stall with a single cell covering about seven blade passages rotating at one fourth of impeller speed opposite to the direction of rotation in the relative frame of reference. Q.H. Nagpurwala [27], [28] carried out experiments to characterize the stalling behavior and suppressing rotating stall using two specially designed jet nozzles injecting high velocity air axially at rotor tip. The work reported in this paper is on the stability & control analysis of a centrifugal compressor stage using Bifurcation method based on MG86 model.

2. BIFURCATION ANALYSIS

2.1 GOVERNING EQUATIONS USED FOR ANALYSIS

The following three non-linear ordinary differential equations have been taken for the analysis that results from the one-mode truncations as the notation in reference [13] known as MG86 model.

$$\Phi'(\xi) = -\Psi + \Psi_c(\Phi) - 3\Phi R \text{-----} (1)$$

$$\Psi'(\xi) = 1/\beta^2 (\Phi - \Phi_T(\Psi)) \text{-----} (2)$$

$$R'(\xi) = \sigma R (1 - \Phi^2 - R) \text{-----} (3)$$

2.2 ASUMPTIONS FOR THE GOVERNING EQUATIONS

1. Flow enters an inlet region where it is assumed to be incompressible and irrotational.
2. Any disturbance that develops in the inlet is assumed to move straight through the compressor.
3. Plenum dimensions are large compared to those of compressor and its ducts so that the velocities and fluid accelerations in the plenum can be considered negligible i.e. all angular variations are lost in the plenum chamber.

The variable Φ is non-dimensional mass flow coefficient which has been shifted so that zero mass flow actually occurs at $\Phi = -1$ and rescaled with W . Ψ is non-dimensional pressure rise of compressor rescaled with H . Both of these variables are averaged over the annulus of the compressor. These rescaling give rise to the parameter S . The parameter β is related to Greitzer 'B' parameter [12], and represents the compressor rotational speed, plenum volume, cross sectional area of compressor and the effective length of the compressor. The function Ψ_c represents the response of the compressor in steady axisymmetric flow.

The throttle characteristic is modeled as

$$\Phi_T = \sqrt{\gamma \Psi} - 1 \text{-----} (4)$$

The cross-sectional area of the throttle is directly proportional to the parameter γ and control can be done by opening and closing the throttle i.e. by increasing or decreasing γ .

For the analysis of the centrifugal compressor, we have taken the three non-linear differential equations 1-3 along with the throttle characteristic equation (4). The compressor characteristic curve has been taken from the experimental result [29]. From the experimental result the compressor characteristic is represented by

$$\Psi_c = \Psi_{c0} + 3.1026 \phi - 8.0135 \phi^2 + 3.3657 \phi^3 \text{---} (5)$$

Where $\Psi_{c0} = 0.225$

The above compressor characteristic is altered by rescaling and used in governing equations.

$$\Psi_c = \Psi_{c0} + 1.5178 + 0.9328 \Phi - 0.5529 \Phi^2 + 0.0322 \Phi^3 \text{-----} (6)$$

Where $\Psi_{c0} = 1.315$. The pressure rise $\Psi_r(\Phi)$ for the rotating stall equilibrium can be seen similar to that of compressor characteristic, as derived below.

$$\Psi_r = \Psi_c - 3 \Phi R$$

$$= \Psi_{c0} + 1.5178 - 2.0672 \Phi - 0.5529 \Phi^2 + 3.0322 \Phi^3 \text{ ----- (7)}$$

2.3 EQUILIBRIUM SOLUTIONS FOR CENTRIFUGAL COMPRESSOR

Setting the left hand side of equations (1) – (3) to zero, and solving the resulting set of coupled nonlinear algebraic equations, equilibrium solutions of MG86 model can be obtained.

Equation (3) reveals two possibilities,

1. Axi-Symmetric Equilibrium (ASE) solutions with $R=0$, which is the compressor characteristic as in equation (6).
2. Rotating Stall Equilibrium (RSE) with $R=1-\Phi^2$

From equation (2), it is seen that, the equilibrium solutions do not depend on β but β influences their stability. Depending on the value of β different types of bifurcation diagrams are possible for the choice of parameters in this study. All have the same equilibrium solutions, but with different stability properties showing different dynamical behavior.

Equilibrium solutions, and the stability of this model are computed numerically by the technique of continuation and bifurcation method. In this method, a continuation algorithm is used to compute all equilibrium states and periodic solutions and their stability by varying throttle parameter γ , for different values of β , each representing one of the possible types of bifurcation diagram. The complete computation of this model in equation (1)- (3), by taking the rescaled value of the compressor characteristic has been carried out by AUTO97 a continuation and bifurcation software by Doedel [30].

2.4 RESULTS AND DISCUSSIONS OF THE MODEL

The bifurcation diagram showing equilibrium values of Ψ vs. γ is plotted in Figure-1 for $\beta=0.2$. The full lines denote stable equilibrium, and dashed lines denote unstable equilibrium. With decreasing γ , i.e. as the mass flow slowly reduced the axisymmetric equilibrium solutions lose stability at a transcritical bifurcation at γ_T , indicated by an open square. At this transcritical bifurcation point a slight perturbation in the mass flow causes the system to enter to the RSE through a jump causing a large drop in pressure rise and mass flow. After entering to RSE, if the throttle is increased then the system under goes through a

saddle node bifurcation at γ_s as indicated and exits rotating stall through another jump. So a hysteresis behavior is found. This hysteresis behavior is well explained in [7], [10] and [14]. At points H2, H1, H3 the system shows some periodic orbits to generate. But as these point H1 and H2 are very close to the Zero mass flow we are deviating from those points and the Hopf bifurcation point H3 occurs for negative value of R which is irrelevant. The equilibrium solutions do not undergo any further bifurcations and thus there are no surge limit cycles for $\beta=0.2$.

Similar plots, for the equilibrium values of R with varying γ is plotted and shown in Figure-2. This plot also shows the drop in pressure rise and mass flow associated with a jump to rotating stall at γ_T and with increasing the throttle again there is a jump to ASE at γ_s . The stable equilibrium for negative R in this figure is physically irrelevant as $R > 0$. Similar graphs can also be plotted easily for the pressure rise Ψ against Φ .

The bifurcation diagrams for $\beta = 0.85$ is shown in Figure-3 and Figure-4 with Ψ plotted against γ and R plotted against γ respectively. When compared with Figure-1 and Figure-2 the rotating stall equilibrium solutions in Figure-3 and Figure-4 shows an additional Hopf bifurcation labeled H2a. Limit cycles born at H2a are nothing but the classic surge oscillation. Limit cycle originating at H1 is nothing but the unstable deep surge, which will ultimately, enters in to rotating stall.

The location of the Hopf bifurcation points H1, H2a, and H2b in the β - γ parameter space can be computed as shown in Figure-5 showing the location of the Transcritical bifurcation at γ_T . It is clearly seen from the graph that with increasing β , the H1 locus approaches the Transcritical bifurcation γ_T but it never touches the Transcritical bifurcation. As described by McCaughan [13], the Hopf bifurcation H2a originates at a taken-Bogdunov bifurcation point. For increasing values of β , H2b occurs for decreasing values of γ with the RSE solution being stable only for the range between H2a and H2b.

From the two parameter continuation β , γ parameter space it is very clear that stable classic surge would occur only for $\beta > 1.27$ i.e. the compressor will enter directly into classic surge at the Transcritical bifurcation point with out entering into rotating stall equilibrium. For no value of β it is seen that the compressor will enter into stable deep surge. Only unstable deep surge can occur in case of a throttle slam, which ultimately falls in to the RSE. For $\beta < 1.27$ if the throttle is varied slowly, the compressor will first enters into RSE.

Classic surge originates as a family of limit cycles that emerge at the Hopf bifurcation of the rotating stall equilibrium solutions labeled H2a in Figure-3 and Figure-4. These limit cycles are tracked with varying γ for $\beta = 0.85$. It is seen that the unstable limit cycles do not under go a fold bifurcation for the value of $\beta = 0.85$, and therefore, stable classic surge oscillations are not created. Instead the limit cycle abruptly terminate at $\gamma = 1.26$. The termination point can be seen to correspond to a saddle loop by observing the rapid increase in the period of oscillation of the limit cycle as they approach the termination point as shown in the Figure-6.

In Figure-7, phase plane displaying a selected orbit 10, Ψ against Φ has been shown. Similar plots R against Ψ and R against Φ have also have been plotted and shown easily. All the orbits are periodic in nature. This clearly shows that classic surge occurs with rotating stall.

3 EFFECTS OF ROTATIONAL DISTURBANCES

In literature it has been mentioned that a rotational disturbance at the inlet to the compressor is to alter the compressor characteristic. This external disturbance can be created by introduction of pre-swirl, use of inlet guide vanes or due to air injection. Agarwal [10] has done this type of analysis for axial flow compressor by introducing a perturbation parameter. This can be understood looking at equation (1) with the terms arranged as follows.

$$\Phi'(\xi) = -\Psi + [\Psi_c(\Phi) - 3\Phi R] \text{-----} (8)$$

In the presence of rotational asymmetry ($R \neq 0$), the bracketed term can be thought to represent a modified compressor characteristic. Similarly, any externally induced rotational disturbance can be modeled as a perturbation to the compressor characteristic.

Here the effect of rotational disturbance is considered in terms of a modified compressor characteristics with a single perturbation term as follows.

$$\Psi_{\text{mod}}(\Phi) = \Psi_c(\Phi) - r\Phi \text{-----} (9)$$

Where $r \geq 0$ is the perturbation parameter. The modified characteristic retains the cubic nature of the original characteristic. The modified characteristic given by equation (9) is used to solve the original equations (1) - (3) with $r = 0.7$.

The resulting bifurcation diagram of Ψ against γ for $\beta = 0.75$ is plotted in Figure-8. It may be observed that the Transcritical bifurcation is shifted to the right and occurs at $\gamma_T \approx 1.57$ at the same time classic surge has

also been avoided. The effect of shifting the transcritical bifurcation point towards the right is to decrease the pressure rise having the same mass flow. By introducing a perturbation parameter of 0.7 we are losing a pressure rise of 0.7, which is highly undesirable.

4 BIFURCATION CONTROL BY Gu.

Linear controllers depend on local stability information and are usually unable to alter the global stability characteristic of the nonlinear systems in a desired manner. A nonlinear feed back control law proposed by Gu is used. This law is based on the principle that the jump phenomenon and the accompanying hysteretic response could be avoided if the sub critical bifurcation at the onset of the rotating stall could be converted into a supercritical one. This requires a feedback that alters the throttle characteristic in equation (4), such that multiple equilibrium states are avoided for a given value of throttle parameter. The law put forward by Gu. [9], requires only feedback of compressor pressure rise Ψ which can be easily measurable is given as:

$$\gamma = (\sqrt{\gamma_0} + K/\sqrt{\Psi})^2 \text{-----} (10)$$

Where γ_0 is the new nominal value of the throttle parameter and K is the controller gain parameter.

This control law changes the shape of the throttle characteristic, however the shape of the compressor characteristic and rotating stall characteristic remains same. By using the above law, the modified throttle characteristic becomes:

$$\Phi_T = \sqrt{\gamma_0 \Psi} + K - 1 \text{-----} (11)$$

And equation (2) becomes:

$$\Psi'(\xi) = 1/\beta^2 (\Phi - \sqrt{\gamma_0 \Psi} - K + 1) \text{-----} (12)$$

In order to eliminate the hysteresis between steady axisymmetric flow and rotating stall, the value of K has to be determined. By the geometric shape of the $\gamma - R$ plot, it is clear that the bifurcated solution at $\gamma = \gamma_T$ can be written as $\gamma_0 = \gamma(R)$, at least for the range where rotating stall is present. The maximum value of $\gamma(R)$ occurs at $R > 0$ and is responsible for jump between RSE characteristic. If this maximum value of γ is made to occur at $R = 0$, then sudden jump can be avoided.

At the optimum point, the equilibrium value of $\Phi_e = 1$, and $R_e = 0$. From the modified system dynamics, we get

$$\Phi_e = \sqrt{\gamma_0 \Psi_e} + K - 1, \text{ and the throttle parameter as}$$

$$\gamma_0 = (\Phi_e - K + 1)^2 / \Psi_e \text{-----} (13)$$

To eliminate hysteresis, maximum value of $\gamma_0(R)$ must occur at $R = 0$.

$$\text{i.e. } d\gamma_0/dR = (d\gamma_0/d\Phi_e)(d\Phi_e/dR) = 0 \text{ -----(14)}$$

We solve for $\Psi_{co} = 1.315$ with $\Phi_e = 1, R = 0$

From, $R = 1 - \Phi^2$, we get,

$$d\Phi_e/dR = -\frac{1}{2} \Phi_e \text{----- (15)}$$

From equation (11),

$$d\gamma_0/d\Phi_e = [2\Psi_e(\Phi_e - K + 1) - (\Phi_e - K + 1)^2 (9.0966\Phi^2 - 1.1058\Phi - 2.0672)]/\Psi_e^2 \text{----- (16)}$$

Substituting the values of equations (15) and (16) in the equation (14), we get, $K = 2$ or, $K = 0.904$. Thus, in order to eliminate the hysteresis between rotating stall and the steady axisymmetric flow the value of K should lie between 0.904 and 2. This can be shown by center manifold theorem [9] that the equilibrium point corresponding to bifurcation value at $\gamma_0 = \gamma_{or}$ for values of K between 0.904 and 2 is locally asymptotically stable. The value of K is taken as 0.904.

The closed loop bifurcation diagram is shown in Figure-9 for $\beta = 0.43$. It is observed from this figure there is super critical bifurcation than sub critical bifurcation as in the case of open loop diagram. In the closed loop, the bifurcated rotating stall solutions ($R > 0$) are all stable eliminating the jump and hysteresis at the point of onset of instability where as in case of open loop it does not. It is seen from $\beta - \gamma_0$ parameter in Figure-10, that beyond $\beta = 0.43$ though the sub critical point is supercritical, it pushes the compressor to enter into surge. This can be seen from the Figure-11, which has been plotted R against γ_0 . Thus this type of control cannot be used for global stability of the system.

4. CONCLUSIONS

The dynamics of centrifugal compressor instabilities rotating stall and surge has been analyzed on the basis of the Moore-Greitzer MG86 model. Experimental data has been taken, and with suitable rescaling of compressor characteristic has been inserted in to the model. With varying system parameters the onset of rotating stall and surge limit cycles have been predicted by using bifurcation method. The experimental results are compared to this analysis. The predictions of which mode of compression system instability, rotating stall or surge will occur are all well in agreement with the experimental data. All the computations have been done using a continuation algorithm for varying throttle parameter γ , for different speed parameter β .

This report clearly shows that deep surge arises from a Hopf bifurcation of the steady axisymmetric flow where as classic surge is born at a Hopf bifurcation of the rotating stall point. A proposed control law has been analyzed in this present study. Study shows that for avoiding hysteresis the control law proposed by Gu. [9] can be used for compressor only upto a certain compressor rotational speed. In the above law the uncertainties due to the approximation of compressor characteristic and throttle characteristic are not considered.

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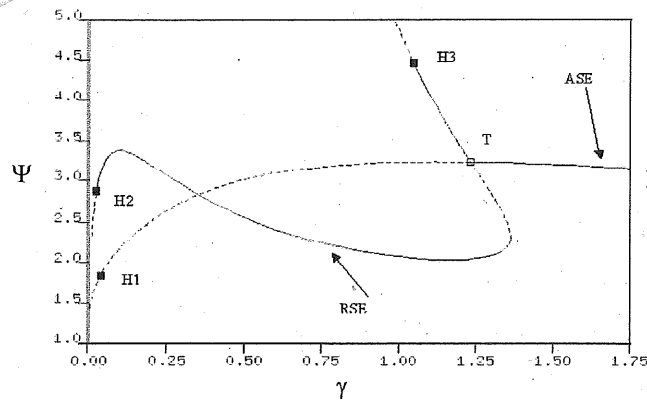


Figure-1 Bifurcation diagram showing equilibrium values of Ψ vs. γ for $\beta=0.2$
Full lines: Stable, Dashed lines: Unstable, Open square: Transcritical, Closed square: Hopf

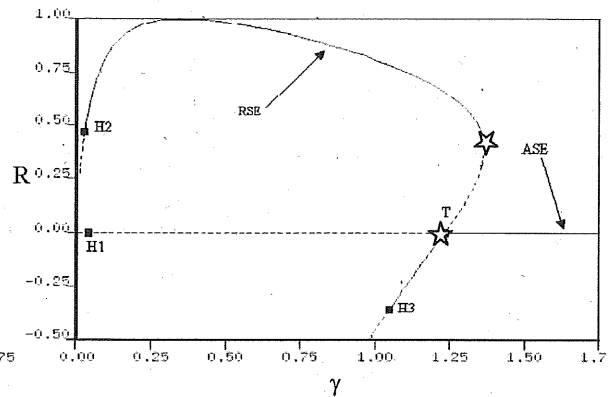


Figure-2 Bifurcation diagram showing equilibrium values of R vs. γ for $\beta=0.2$
Full lines: Stable, Dashed lines: Unstable, Open square: Transcritical, Closed square: Hopf

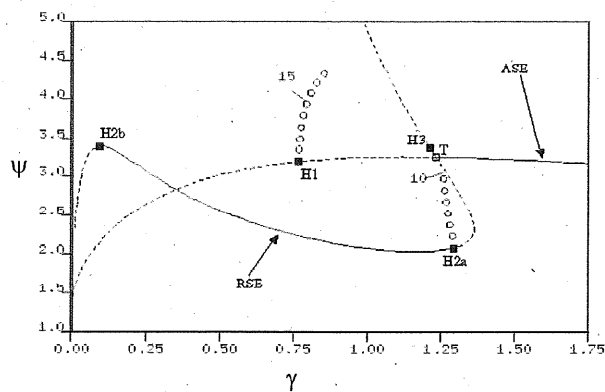


Figure-3 Bifurcation diagram showing equilibrium values of Ψ vs. γ for $\beta=0.85$
Full lines: Stable, Dashed lines: Unstable, Open square: Transcritical, Closed square: Hopf

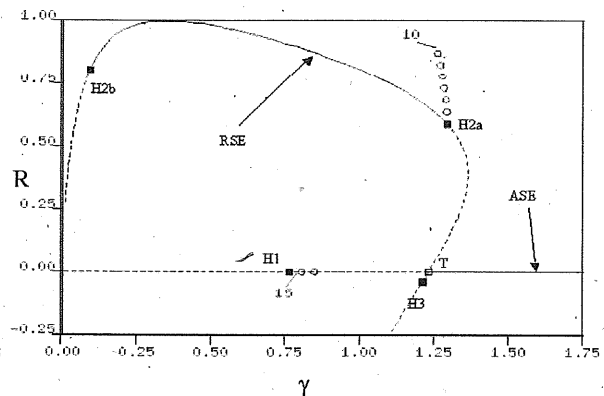


Figure-4 Bifurcation diagram showing equilibrium values of R vs. γ for $\beta=0.85$
Full lines: Stable, Dashed lines: Unstable, Open square: Transcritical, Closed square: Hopf

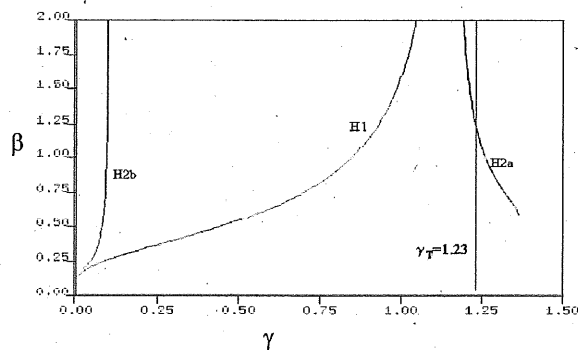


Figure-5 Bifurcation diagram showing two parameter continuation values of β vs. γ

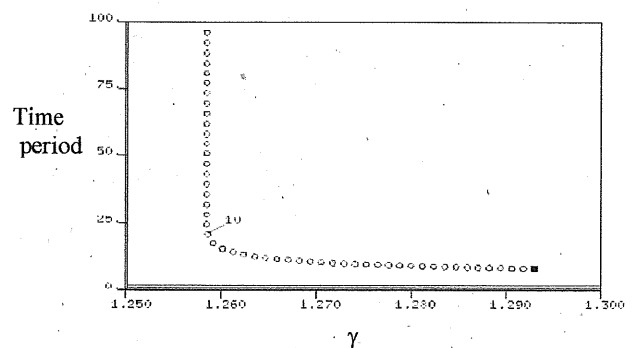


Figure-6 Bifurcation diagram showing period limit cycles vs. γ for $\beta=0.85$ in classic surge.
Closed square: Hopf, open circle: Unstable

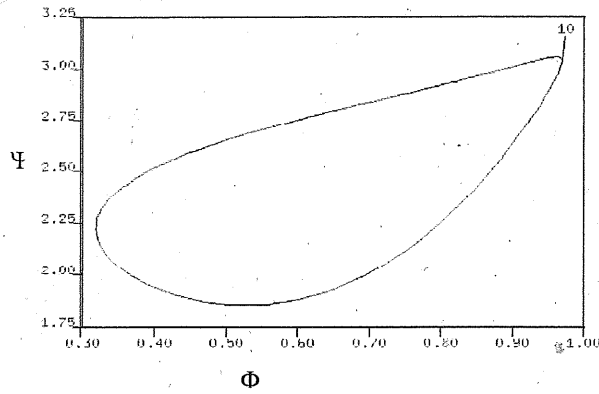


Figure-7 Phase plane displaying a selected labeled orbit 10 Ψ vs. Φ for $\beta=0.85$ in classic surge

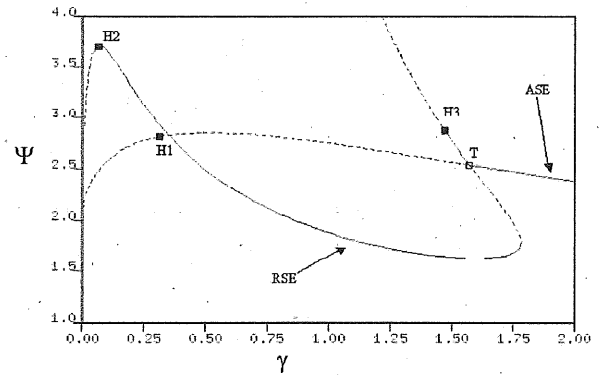


Figure-8 Bifurcation diagram showing equilibrium values of Ψ vs. γ for $\beta=0.75$, with perturbation parameter $r=0.7$

Full lines: stable, Dashed lines: Unstable, Closed square: Hopf, Open square: Transcritical

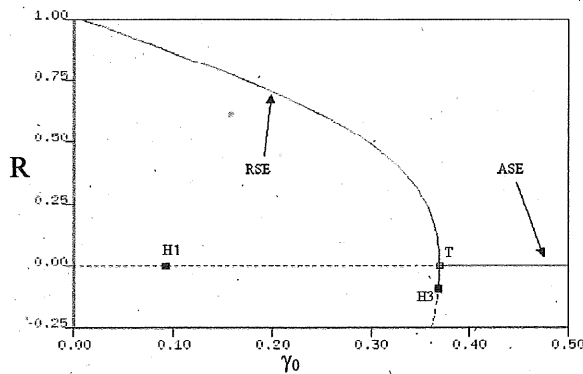


Figure-9 Bifurcation diagram showing equilibrium value of R vs. γ_0 with control (law proposed by Gu.) with $K=0.904$ for $\beta=0.43$ in a closed loop system

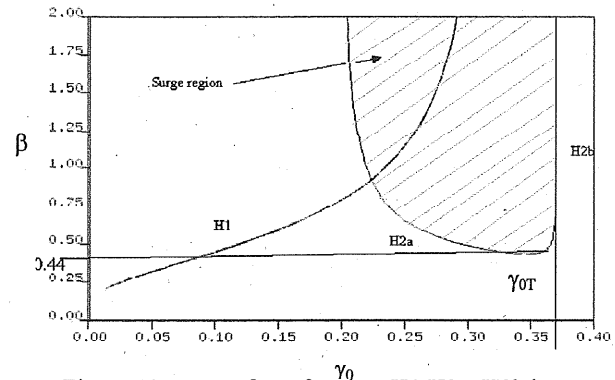


Figure-10 Locus of Hopf points $H1, H2a, H2b$ in two parameters continuation with varying parameters γ_0 and β for the closed loop compressor system with the control law proposed by Gu. Here $K=0.904$

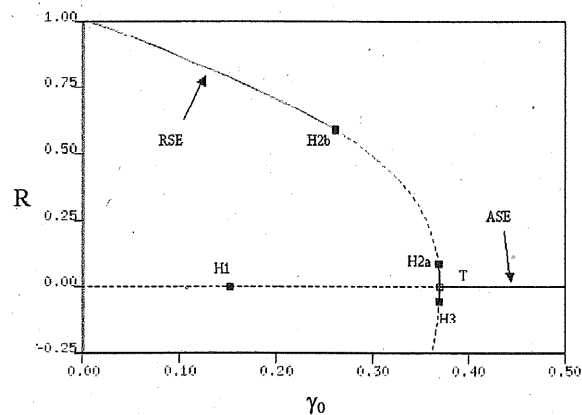


Figure-11 Bifurcation diagram showing equilibrium value of R vs. γ_0 with control (law proposed by Gu.) with $K=0.904$ for $\beta=0.6$ in a closed loop system

EXTENDED ABSTRACT

Stability and Control Analysis of Centrifugal Compressor

Manas Pradhan**, S. Ramamurthy* and V.K. Mayavanshi**

Compressor stall and surge are complex nonlinear instabilities that reduce the performance and can cause failure of the system. The emphasis has been on gaining a better insight to stall/surge dynamics and to predict their occurrence and controlling. This paper presents a bifurcation-based analysis of the dynamical behavior and control of stall and surge in centrifugal compressor stage. The experimentally measured compressor characteristics and MG86 model given by Greiter and Moore was used for stability analysis. The experimental results are compared with this analysis. The predictions of compressor system instability, rotating stall and surge are all well in agreement with the experimental data. Analysis gives compressor instabilities and provides a complete picture of the flow behavior at different operating points, which can be seen from the Speed (β) and Throttle opening (γ) parameters as shown in Figure-1. The location of the Hopf bifurcation points H_1 , H_{2a} , and H_{2b} in the β - γ parameter curve is also shown in this Figure. The location of the Transcritical bifurcation in this figure is marked as γ_T . It is clearly seen from this figure with increasing β , the H_1 locus approaches the Transcritical bifurcation γ_T but it never touches the Transcritical bifurcation. As described by McCaughan the Hopf bifurcation H_{2a} originates at a Taken-Bogdunov bifurcation point. For increasing values of β , H_{2b} occurs for decreasing values of γ , with the RSE solution being stable only for the range between H_{2a} and H_{2b} . From the two parameter continuation β , γ parameter curve it is very clear that stable classic surge would occur only for $\beta > 1.27$ i.e. the compressor will enter directly into classic surge at the Transcritical bifurcation point without entering into rotating stall equilibrium (RSE). For no value of β it is seen that the compressor will enter into stable deep surge. Only unstable deep surge can occur in case of a throttle slam, which ultimately falls in to the RSE. For $\beta < 1.27$ if the throttle is varied slowly, the compressor will first enter into RSE.

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A proposed nonlinear control law by Gu along with the effect of rotational disturbances has been studied using this method. The effect of rotational disturbances has been studied by introducing a perturbation parameter and seen that classic surge can be avoided at the cost of decrease in pressure rise. Thus the value of the perturbation parameter should be as low as possible. Using control law, by Gu it was shown that, abrupt entry of compressor into rotating stall could be avoided, eliminating the hysteresis between entry into and recovery from the rotating stall only up to a certain value of compressor rotational speed.

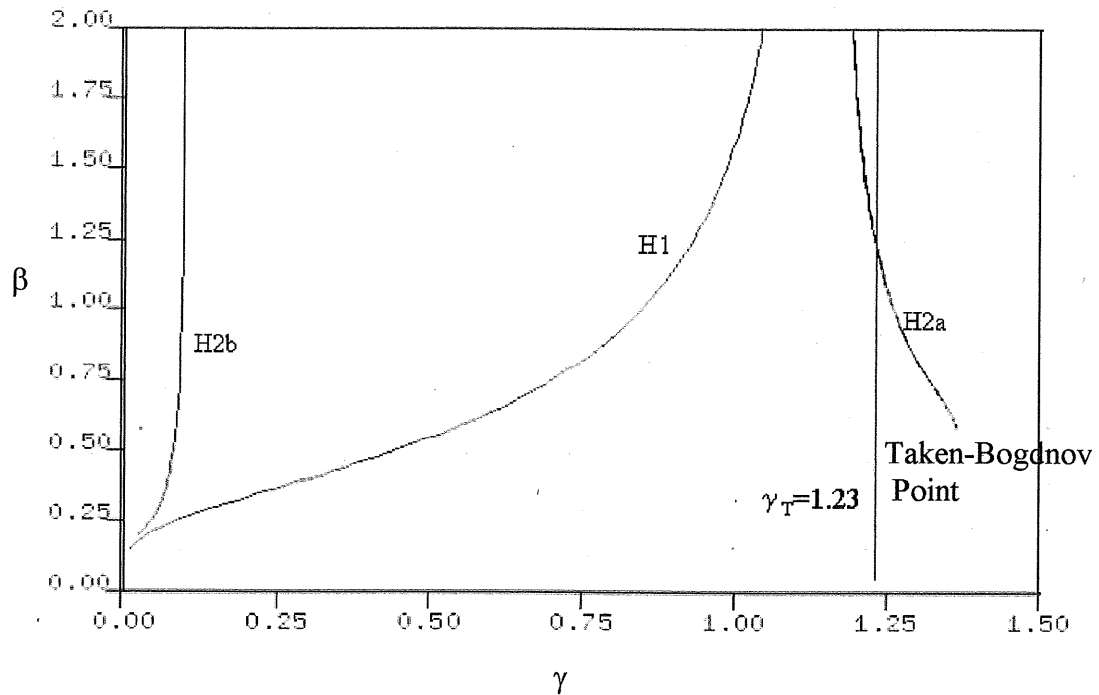


Figure-1 Bifurcation diagram showing equilibrium values of β vs. γ for $\beta=0.85$
 Full lines: Stable, Dashed lines: Unstable, Open square: Transcritical, Closed square: Hopf